The Mathletes were introduced to the golden ratio, also known as the divine proportion, golden mean, or golden section, is a number often encountered when taking the ratios of distances in simple geometric figures such as the pentagon, the human body, art, architecture, nature, space, and much more. It is identified by the Greek symbol Phi Φ.

All Mathletes should try the challenge on pages 3, 6 and 7 of this file. Younger Mathletes should ask their parents to work with them. Older Mathletes may want to involve their parents as well as this is fascinating material.

This ratio is approximately 1:1.618033989. Why approximately? Because the number Phi is irrational which means the decimals go on forever without terminating or repeating. Other examples of irrational numbers are pi or π which is approximately, 3.141592653....... and the square root of 2 which is approximately 1.414213562.... Rational numbers are numbers like 5, -2, 5.25, 1/3 because .33 repeats.

Let us first explore, what is a ratio?

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:). Suppose we want to write the ratio of 8 and 12.
We can write this as 8:12 or as a fraction 8/12, and we say the ratio is *eight to twelve*.
Examples:
Jeannine has a bag with 3 videocassettes, 4 marbles, 7 books, and 1 orange.
1) What is the ratio of books to marbles?
Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be 7/4.
Two other ways of writing the ratio are 7 to 4, and 7:4.
2) What is the ratio of videocassettes to the total number of items in the bag?
There are 3 videocassettes, and 3 + 4 + 7 + 1 = 15 items total.
The answer can be expressed as 3/15, 3 to 15, or 3:15.

Younger Mathletes may be confused by the number 1.618033989. What is this decimal thing? Even though we discussed it in Mathletes, we should take time to discuss it now.

1.5 is the decimal expression for 1 and 1/2 and Phi is a little more than 1.5.

Decimal Numbers

Decimal numbers such as 3.762 are used in situations which call for more precision than whole numbers provide.

As with whole numbers, a digit in a decimal number has a value which depends on the place of the digit. The places to the left of the decimal point are ones, tens, hundreds, and so on, just as with whole numbers. This table shows the decimal place value for various positions:

Note that adding extra zeros to the right of the last decimal digit does not change the value of the decimal number.

Place (underlined)	Name of Position
1.234567	Ones (units) position
1.234567	Tenths
1.234567	Hundredths
1.234567	Thousandths
1.234567	Ten thousandths
1.234567	Hundred Thousandths
1.234567	Millionths

Example:

In the number 3.762, the 3 is in the ones place, the 7 is in the tenths place, the 6 is in the hundredths place, and the 2 is in the thousandths place.

Example:

The number 14.504 is equal to 14.50400, since adding extra zeros to the right of a decimal number does not change its value.

We will go over this during Mathletes in the coming weeks.

The best way to describe discover the number Phi is to explore the Fibonacci sequence. ***(see page 4 for the relationship between the Fibonacci sequence and the golden ratio).*** **The series begins with 0 and 1. After that, use the simple rule:**

Add the last two numbers to get the next.

0,1,1, 2, 3, 5, 8, 13, 21

The big challenge for the week:

How many Fibonacci numbers can you find? The first 50 Fibonacci numbers are on the next page. I also have included the prime factors of each Fibonacci number.

The first 300 Fibonacci numbers

0 : 0
1 : 1
2 : 1
3 : 2
4 : 3
5 : 5
6 : 8
7 : 13
8 : 21
9 : 34
10 : 55
11 : 89
12 : 144
13 : 233
14 : 377
15 : 610
16 : 987
17 : 1597
18 : 2584
19 : 4181
20 : 6765
21 : 10946
22 : 17711
23 : 28657
24 : 46368
25 : 75025
26 : 121393
27 : 196418
28 : 317811
29 : 514229
30 : 832040
31 : 1346269
32 : 2178309
33 : 3524578
34 : 5702887
35 : 9227465
36 : 14930352
37 : 24157817
38 : 39088169
39 : 63245986
40 : 102334155
41 : 165580141
42 : 267914296
43 : 433494437
44 : 701408733
45 : 1134903170
46 : 1836311903
47 : 2971215073
48 : 4807526976
49 : 7778742049
50 : 12586269025

How do we take the Fibonacci sequence and find the golden ratio?

If we take the ratio of two successive numbers in Fibonacci's series, (1, 1, 2, 3, 5, 8, 13, ..) and we divide each by the number before it, we will find the following series of numbers:
1/1 = 1, 2/1 = 2, 3/2 = 1·5, 5/3 = 1·666..., 8/5 = 1·6, 13/8 = 1·625, 21/13 = 1·61538...

The ratio seems to be settling down to a particular value, which we call **the golden ratio** or **the golden number**. It has a value of approximately **1·618034.**

The **golden ratio** 1·618034 is also called the **golden section** or the **golden mean** or just the **golden number**. It is often represented by a greek letter **Phi**. The closely related value which we write as **phi** with a small "p" is just the decimal part of Phi, namely 0·618034.

This is what happens if we take the ratios the other way round i.e. we divide each number by the one *following* it: 1/1, 1/2, 2/3, 3/5, 5/8, 8/13,

If you feel comfortable with long division, find Phi ≈(1.618033989)by dividing the largest Fibonacci number you found by the Fibonacci number before it. The symbol ≈ means approximate.

Or find phi ≈(.618033989)by dividing any Fibonacci number you found by the Fibonacci number after it.

Find Golden Ratios Everywhere

In Mathletes class, we divided the measurement of our height by the distance from our toes to our belly button (navel). All of us came within one tenth of the Golden Ratio -- Phi; some even closer. Of course, the measurements we took were not precise. Taking measurements in centimeters may be more accurate.

Mr. Kramer's height is 69 inches and the distance from my toes to navel is 40 inches giving me a ratio of 69/40 of 1.725. My ratio differs from the golden ratio by a little more than a tenth or 107 one thousandths to be exact.

The next few pages go through pictures and discussions of other Golden Ratios found in the universe. Find some on your own.

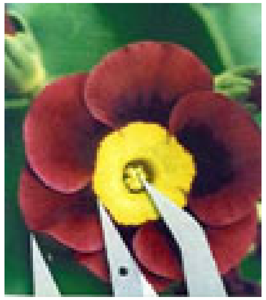
Looking For Beauty

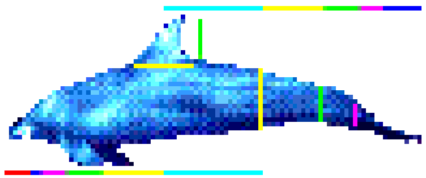
The Greeks said that **all beauty is mathematics**. If that is true then perhaps there is a mathematical code, formula, relationship or even a number that can describe facial beauty.

Historically many different numbers have been tried in an attempt to describe beauty however, only one mathematical relationship has been consistently and repeatedly reported to be present in beautiful things.

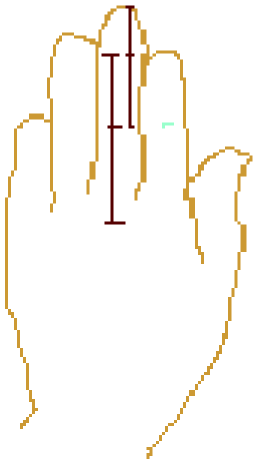
These pictures are taken from the work of Dr. Steven Marquardt, a surgeon who does research into human attractiveness. The Mathletes saw a 7 minute clip of a documentary narrated by John Cleese about Marquardt's research. The instrument you see here is a golden divider (it measures lengths of 1.618 to 1). I am trying to acquire one of these instruments.

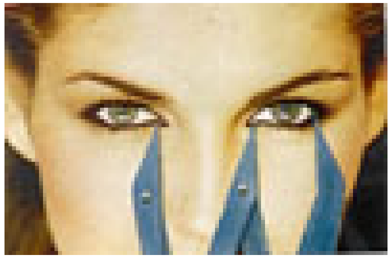
In Nature

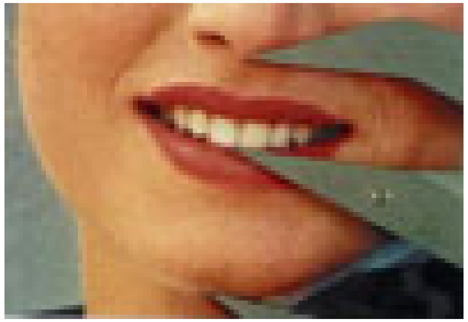


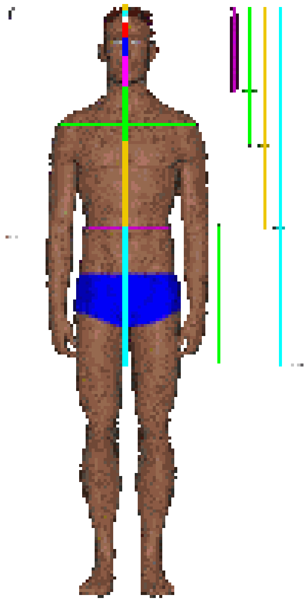


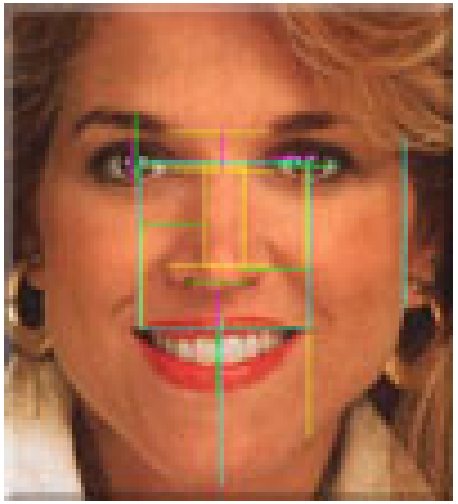
In the Human Body



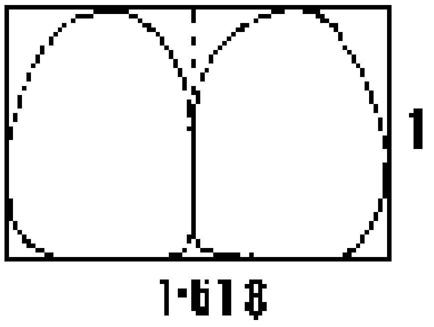






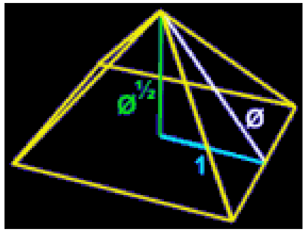


In the Human Teeth

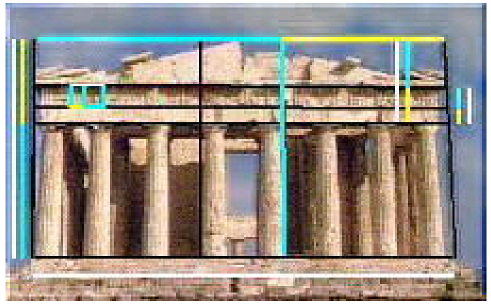


In Architecture

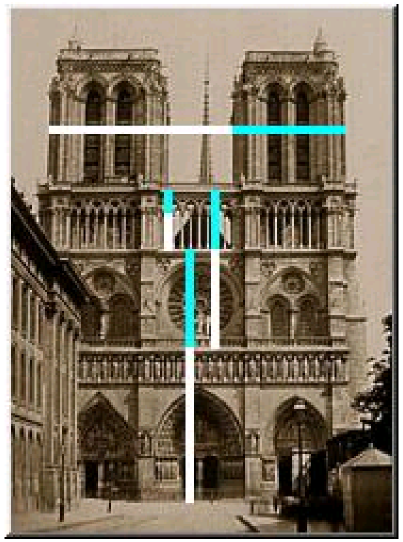
the Pyramids in Eqypt



the Parthenon in Athens, Greece

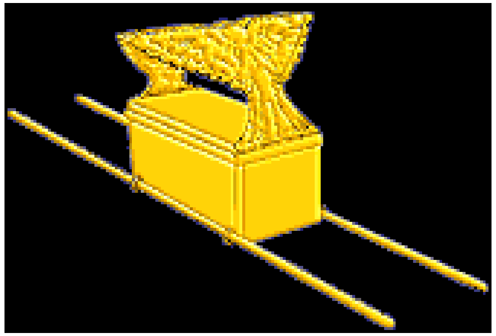


Notre Dame in Paris, France



the United Nations building in New York, NY

In the Bible



In Exodus 25:10, God commands Moses to build the Ark of the Covenant, in which to hold His Covenant with the Israelites, the Ten Commandments, saying,

"Have them make a chest of acacia wood-
two and a half cubits long,
a cubit and a half wide,
and a cubit and a half high."

The ratio of 2.5 to 1.5 is 1.666..., which is as close to phi (1.618...) as you can come with such simple numbers and is certainly not visibly different to the eye. The Ark of the Covenant is thus constructed using the Golden Section, or Divine Proportion. This ratio is also the same as 5 to 3, numbers from the Fibonacci series.

Note: A cubit is the measure of the forearm below the elbow.

Noah's Ark uses a Golden Rectangle

In Genesis 6:15, God commands Noah to build an ark saying,

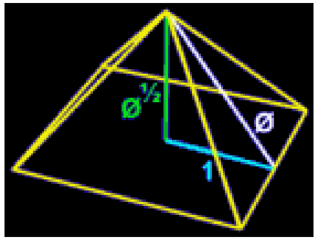
"And this is the fashion which thou shalt make it of: The length of the ark shall be three hundred cubits, the breadth of it fifty cubits, and the height of it thirty cubits."

Thus the end of the ark, at 50 by 30 cubits, is also in the ratio of 5 to 3, or 1.666..., again a close approximation of phi not visibly different to the naked eye. Noah's ark was built in the same proportion as ten arks of the covenant placed side by side.

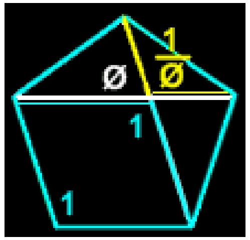
In Credit Cards

Standard sized credit cards are 54mm by 86mm, creating a ratio of 0.628, less than a millimeter off from a perfect golden section of 0.618

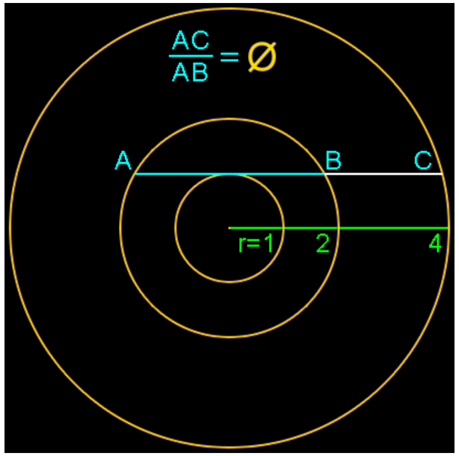
In Geometry



Phi or Φ, was described by Johannes Kepler as one of the "two great treasures of geometry." (The other is the Theorem of Pythagoras.)



In a triangle it forms the dimensions of the great pyramids of Egypt. A ruler and compass can be used to form the "golden rectangle" used by the Greeks in the Parthenon. (See also the Orthogons page.) Phi also defines the dimensions of a pentagon.

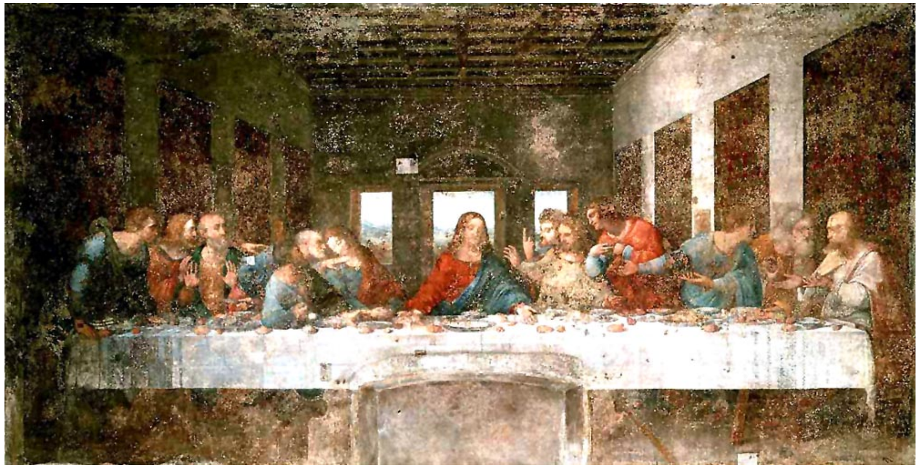


Phi can be constructed from concentric circles whose sizes are in a ratio of 1 : 2 : 4. Draw a tangent from the small circle through the other two and the ratio of AC to AB is Φ, as AB = 2 * 3 ½ and AC = 15½ + 3½, which by factoring out the 3½ can be reduced to a ratio of 2 to (5½+1), or Φ.

In History (Leonardo da Vinci) -- The Divine Proportion

Da Vinci provided illustrations for a dissertation published by Luca Pacioli in 1509 entitled "De Divina Proportione" (1), perhaps the earliest reference in literature to another of its names, the "Divine Proportion." This book contains drawings made by Leonardo da Vinci of the five Platonic solids. It was probably da Vinci who first called it the "sectio aurea," which is Latin for golden section.

The Renaissance artists used the Golden Mean extensively in their paintings and sculptures to achieve balance and beauty. Leonardo Da Vinci, for instance, used it to define all the fundamental proportions of his painting of "The Last Supper," from the dimensions of the table at which Christ and the disciples sat to the proportions of the walls and windows in the background.



In all aspects of the universe:

energy
music
Penrose Tiling
population growth
quasi-crystals
solar system
spirals
stock markets

****for more information go to my favorite web site for the Golden Ratio http://goldennumber.net.htm.***